

Discrete Math

7th and 8th

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#GoBears



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Executive Summary

Day 1-3: WHAT'S YOUR NUMBER?

Day 4-6: ROCK, PAPER, SCISSORS

Day 7-8: BUILD A BURGER!

Day 9-10: WHO WANTS TO BE A MILLIONAIRE?

Day 11-12: BLACK JACK!

Day 13-14: SHAKE ON IT!

Day 15-16: ROAD TRIP!

Description:

- (1) Students will use the counting principle to determine how many combinations are possible for the following activities.
- (2) Students will make sample spaces of potential wins and loss situations for a best of 3 situation. They will use tree diagrams to check their answers. Based on their results students will calculate how many different ways there are to win a best of 3 tournament. Students then will use the combination formula to calculate the number of ways you could win a best of 5 and best of 7 tournament.
- (3) Students will calculate the number of outcomes are possible given the choice of four different toppings. Students will create a sample space, use a tree diagram to check sample space correctness, and use permutation and combination formulas to find the possible number of outcomes.
- (4) Students will determine how many possible tickets exist in the lottery using the concepts of permutations and combinations. Students will work in groups to develop a scheme to ensure winning the lottery and be able to explain and justify their scheme to the class.
- (5) Students will explore the concept of probability while making the use of counting techniques to figure out the odds to winning the game of BlackJack. This lesson is designed to help student discover the odds of specific pairs of cards being drawn out of a deck of 52 cards.
- (6) During this lesson, students will explore the handshake problem, a classic problem in mathematics that asks, "How many handshakes occur when n people shake hands with each other?" Groups work to determine how many handshakes take place among the nine Supreme Court justices, and then generalize to the number of handshakes in any size group. Students explore the problem using a verbal description, a table, a graph, a picture, and an algebraic formula.
- (7) Students will plan a road trip, starting in Cleveland, to visit friends in Cincinnati, Pittsburgh, Baltimore, and Boston. However, with the price of gas over \$2 a gallon, they will figure out the shortest travel route to save on expenses. This lesson investigates two different methods to determine the shortest route: the Nearest Neighbor method and the Greedy Method.

Lesson 1: What's Your Number? (2-3 days)

Adapted from Illuminations: <http://illuminations.nctm.org/Lesson.aspx?id=4038>

MN Math Standards:

7.4.3.2 Calculate probability as a fraction of sample space or as a fraction of area. Express probabilities as percents, decimals and fractions.

Pre-Test:

- 1) How many different ways can you order the digits 1,2,3 without repeating the numbers?
- 2) How many different ways can you order the digits 1, 2, and 3 with the possibility of repeating the numbers?

Objectives:

Students will use the counting principle to determine how many combinations are possible for the following activities.

Activity 1 : Create a House Number

LAUNCH:

Ask students to share what their house number is or what their address is if they live out in the country. Get them looking at the number of different ways numbers are assigned. (One side of the road is odd the other is even)

EXPLORE/SHARE:

Put students into groups and have them work through activity sheet on creating a house number.

<http://illuminations.nctm.org/uploadedFiles/Content/Lessons/Resources/6-8/AddressNumbers-AS-Grades68.pdf>

Explain to students that they will be forming a three-digit house number using the digits 0-9.



Give students number tiles 0 through 9, or have them cut the digits from the bottom of the activity sheet. Review the problem by reading it to the students. Then, give them some time to explore the problem. Lead a discussion to arrive at a conclusion.

After the discussion, ask students to consider how many more numbers could be formed with a product of 24 if the digits do not have to be different. That is, what happens if a digit can be used more than once in a house number?

SUMMARIZE:

Discuss the tree diagrams for this problem. Tree diagrams are a useful tool for organizing data and finding the amount of outcomes for a problem. You can also point that the counting principle can also be used to solve this problem.

Activity 2: Create a Mailbox Number

Explain to students that they will be forming a five-digit mailbox number using the digits 0-9.



Explore/Share:

Distribute one copy of the Create a Mailbox Number Activity Sheet to each student.

<http://illuminations.nctm.org/uploadedFiles/Content/Lessons/Resources/6-8/MailboxNumber-AS-Grades68.pdf>

Have students work in groups and give them number tiles 0 through 9, or have them cut the digits from the bottom of the activity sheet.

Give student some time to explore the problem.

Have students share their solutions.

Summarize:

Using “bins” and multiplication helped us solve this problem. This method of counting is called the “counting principle” and is useful in many situations when the number of outcomes is a large number or when the number of bins is larger than three.

Extensions:

1. *Activity: Create a House Number*— How many house numbers can be formed if the product of the digits in a four-digit address is 24?

[64 house numbers can be formed with the various combinations of 6221, 6141, 2431, 1831, and 2223.]

2. *Activity: Create a Mailbox Number*— How many five-digit address numbers can be formed if the first two digits form a two-digit cube number and the last three digits form a three-digit cube number less than 400?

[6 mailbox numbers can be formed: 27125, 27216, 27343, 64125, 64216, and 64343.]

3. This activity can also be adapted for license plate numbers for any state.

Post-Test:

- 1) How many different ways can you order the digits 1,2,3 without repeating the numbers?
- 2) How many different ways can you order the digits 1, 2, and 3 with the possibility of repeating the numbers?

LESSON 2: ROCK, PAPER, SCISSORS! (2-3 Days)

Adapted from class notes: Math 6200, Bemidji State University, Summer 2016 also from <http://www.math.ccsu.edu/mitchell/mtmsmathematicalexplorationsafairgame.pdf>

By Cheryl Nelson and Nicole Williams

MN Math Standards:

7.4.3.2 Calculate probability as a fraction of sample space or as a fraction of area. Express probabilities as percents, decimals and fractions.

Pre-Test:

1) If you played a best of 3 tournament, how many different ways could you win?

Objectives:

Students will make sample spaces of potential wins and loss situations for a best of 3 situation. They will use tree diagrams to check their answers. Based on their results students will calculate how many different ways their are to win a best of 3 tournament. Students then will use the combination formula to calculate the number of ways you could win a best of 5 and best of 7 tournament.

Activity: Play a best of 3 game of Rock, Paper, Scissors.

Launch:

Ask students where they have seen a best of 3, 5, 7 tournament. Ask them how many ways their are to win a tournament like this.

Explore/Share:

1) Have students play a number of rounds of “Rock, Paper, Scissors” in partners or groups of three and keep track of how they win (for example: WW, LWW, WLW...etc.).

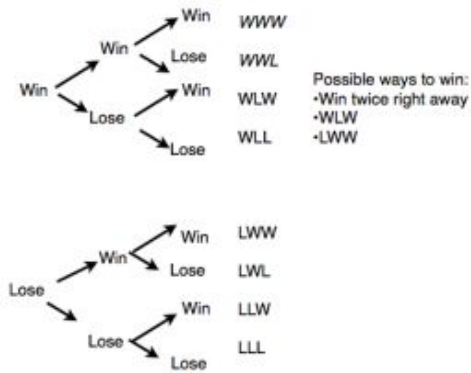
Use the following chart to help students keep track of their game results:

		Player A		
		Rock	Paper	Scissors
Player B	Rock			
	Paper			
	Scissors			

2) Discuss the class findings as a class and develop a list of the sample space. Ask students “how can we be sure we have all of the possible combinations?”

3) Have students work in groups to come up with a method for being 100% sure all possibilities are accounted for. (If students struggle here, hint toward a tree diagram).

4) Allow time for students to share their group's thoughts. Display answers on the board (students should have a total sample space of eight outcomes, but after eliminating duplicate samples, only three different ways to win)



Summarize:

We made a sample space of possible outcomes and found there were eight outcomes. Why are there only three ways to win? (Discuss why WWW and WWL are really the same thing).

Motivate students toward discovery of the combination formula:

$$\binom{n}{r} \text{ or } {}_n C_r = \frac{n!}{r!(n-r)!}$$

- n = number of choices (in a best of three, n = 3. For best of 5, n = 5).
 - r = number of choices we are interested in (in this case, win or loose = 2 choices).
 - n! = sorts all n items into “bins”
 - r! = takes out duplicates (when order doesn't matter)
 - (n-r)! = the number of “bins” that are not being used Discuss the formula and where these numbers come from.
- Why divide?

Continuation:

Now repeat the process for a best of 5 series then a best of 7 series

Post Test:

- 1) How many different ways could you win a best of 3 tournament?
- 2) How many different ways could you lose a best of 5 tournament?

LESSON 3: BUILD A BURGER! (2 Days) (This is actually a lunch menu at our school)

Adapted from class notes: Math 6200, Bemidji State University, Summer 2016

MN Math Standards:

7.4.3.2 Calculate probability as a fraction of sample space or as a fraction of area. Express probabilities as percents, decimals and fractions.

Pre Test:

- 1) How many different ways could I put 3 toppings on a hamburger?

Objectives:

Students will calculate the number of outcomes are possible given the choice of four different toppings.

Students will create a sample space, use a tree diagram to check sample space correctness, and use permutation and combination formulas to find the possible number of outcomes.

Launch:

Find a funny hamburger video. Launch your class into a discussion of hamburgers and what makes a good burger. Lead into a discussion of what toppings they like, vote on four toppings to put on your burger. Given these 4 toppings, discuss how many different burgers you could build.

Explore/Share:

- 1) After posing the question, allow students time to ponder and attempt the problem (students will have had practice with sample space, tree diagrams and the counting principle previous to this lesson). Partner students or group students of small groups of three. Ask students to work together to solve the problem and put their thoughts on a student whiteboard to share with the class.

- 2) Compare student answers. Sample space (lists) and tree diagrams should yield 16 different hamburger combinations. (With tree diagrams, start with the burger then add one topping at a time. The first “bin” will have, for example, ketchup or no ketchup.) A counting principle will also work: $2 \cdot 2 \cdot 2 \cdot 2 = 16$ combinations.

- 3) Use the tree diagram to discuss the following questions:

- How many ways can I start with four toppings, and choose only one?
- How many ways can I start with four toppings, and choose two?
- How many ways can I start with four toppings, and choose three?
- How many ways can I start with four toppings, and choose four?

- 4) Discuss the outcomes. Since the order of the toppings doesn't matter (ketchup, onion is the same as onion, ketchup), we eliminate some of the outcomes.

5) Have students work in partners to work this observation into a formula (the combination formula:

$$\binom{n}{r} \text{ or } {}_n C_r = \frac{n!}{r!(n-r)!}$$

where n = number of choices (4) and r = how many toppings we are choosing. 21

6) Discuss where the numbers in the numerator and denominator are coming from. • The numerator is the factorial of the number of choices available...similar to the counting principle. • The denominator is a product of two things: how many bins we are filling ($r!$) and what we need to eliminate because of duplicate answers $(n-r)!$.

7) Alter the problem: If order IS important (ketchup, onion is a different choice than onion, ketchup), how does that change your answer?

- Choose 0 toppings = $(4!)/(4-0)! = 1$ way to order no toppings
- Choose 1 toppings = $(4!)/(4-1)! = 4$ ways to order 1 topping
- Choose 2 toppings = $(4!)/(4-2)! = 12$ ways to order 2 toppings
- Choose 3 toppings = $(4!)/(4-3)! = 24$ ways to order 3 toppings
- Choose 4 toppings = $(4!)/(4-4)! = 4! = 24$ ways to order 4 toppings
(remember that $0! = 1$)

There is a total of 65 outcomes possible.

Summarize:

When order of the variables (in this case the toppings) does not matter (the order of variables can be reversed, and the outcome is the same), we calculate the outcomes using a combination method or formula. When the order is important (changing the order of the variables yields a new outcome), we calculate the outcomes using a permutation formula and do not cancel out duplicate outcomes.

Extension:

Have students develop their problem using any sort of food item where toppings can be added: Pizza, Ice Cream etc.

Post Test:

- 1) How many different ways can I put 5 toppings on a hamburger if the order did not matter?
- 2) How many different ways can I put on 5 toppings on a hamburger if the order did matter?

LESSON 4: WHO WANTS TO BE A MILLIONAIRE? (2 Days)

Adapted from: The Discrete Math Project at the University of Colorado at Boulder
<http://www.colorado.edu/education/DMP/activities/counting/olcact04.html>

MN Math Standard:

7.4.3.2 Calculate probability as a fraction of sample space or as a fraction of area. Express probabilities as percents, decimals and fractions.

Pre Test:

- 1) If you had to pick 3 numbers between 1 and 24, in correct order using each number only one time, how many different ways could you pick the numbers.

Objective:

Students will determine how many possible tickets exist in the lottery using the concepts of permutations and combinations. Students will work in groups to develop a scheme to ensure winning the lottery and be able to explain and justify their scheme to the class.

Launch:

Find a video of the game show who wants to be a millionaire? Ask if there would be an easier way to be a millionaire - (lottery). Have a discussion on the ways that numbers could be picked. This activity focuses on the application of permutations and combinations a actual lottery. Students are asked to individually determine the total possible tickets and then work in groups to develop a scheme to guarantee winning the lottery. It is intended to be used after the students have been introduced to the concepts of counting, permutations and combinations.

Tell the students that even though there are many people who play the lottery because they feel "lucky", few actually examine the chances that they will actually win. Let them know that they will be determining a method for guaranteeing that they will win the lottery regardless of the jackpot.

Explore/ Share:

Have groups of students work through the worksheet.

<http://www.colorado.edu/education/DMP/activities/counting/olchnd04.html>

Students must be able to explain their answers to the rest of the class.

Summarize:

Discuss with the class which method they preferred after listening to all of the presentations.

Extension:

Have student ask their parents about any types of lottery tickets they purchase. Get some tickets at a local gas station and find the total number of possibilities for winning with the class and discuss the outcome.

Post Test

- 1) If you had to pick 4 numbers between 1 and 24, in correct order using each number only one time, how many different ways could you pick the numbers.

LESSON 5: BLACK JACK! (2 Days)

Adapted from: The Discrete Math Project at the University of Colorado at Boulder

<http://www.colorado.edu/education/DMP/activities/counting/ewsact05.html>

MN Math Standard:

7.4.3.2 Calculate probability as a fraction of sample space or as a fraction of area. Express probabilities as percents, decimals and fractions.

Pre Test:

- 1) If I have a deck of cards, what is the probability of drawing out a King?
- 2) What is the probability of drawing out a King of Spades?

Objective:

Students will explore the concept of probability while making the use of counting techniques to figure out the odds to winning the game of BlackJack. This lesson is designed to help student discover the odds of specific pairs of cards being drawn out of a deck of 52 cards.

Launch:

Ask student if they have ever played Black Jack. Find a video of Black Jack being explained is even better. Explain the game to them. Have them play a the game with betting chips in groups of 4 with one person being the dealer. It may take a little time for kids to understand the game but they will catch on quickly.

Simplified Rules of Blackjack

1. There is a dealer and from one to seven players at a table.
2. A deck of cards is shuffled and cut (several decks of cards may be used). The dealer deals two cards to each player and two cards to himself one face up and one face down.
3. Each player in turn may request one or more additional cards, with the goal of attaining a total value as close to 21 as possible without going over. A player immediately loses if he or she goes over 21.
4. When all players are satisfied with their current totals, the dealer may take additional cards according to the following rules: the dealer must hit (take another card) if the current total is 16 or less, and the dealer must stand (not take another card) if the current total is 17 or more.
5. Any player with a total greater than the dealers total wins. If the dealer goes over 21, all players with a total of 21 or less win.

Explore/Share:

Distribute the “Black Jack” activity sheet and have the students work through the sheet in their groups.

<http://www.colorado.edu/education/DMP/activities/counting/ewshnd05.html>

Have students report their findings to the rest of the class via the board. Discuss the results of the probability that students found.

Summarize:

Have students share any ideas of ways to improve their chances of winning with the results they came up with. Now have students play Black Jack using the rules that they came up with.

Post Test:

- 1) If I have a deck of cards, what is the probability of drawing out a King?
- 2) What is the probability of drawing out a King of Spades?

LESSON 6: SHAKE ON IT! (2 Days)

Adapted from class notes: Math 6200, Bemidji State University, Summer 2016

And from Illuminations website: <http://illuminations.nctm.org/Lesson.aspx?id=2112>

MN Math Standards:

8.2.2.4 Represent arithmetic sequences using equations, tables, graphs and verbal descriptions, and use them to solve problems.

8.2.2.5 Represent geometric sequences using equations, tables, graphs and verbal descriptions, and use them to solve problems.

Pre Test:

- 1) If there are 5 people in a room and they all shook hands with each other, how many handshakes would there be in total?

Objective:

During this lesson, students will explore the handshake problem, a classic problem in mathematics that asks, "How many handshakes occur when n people shake hands with each other?" Groups work to determine how many handshakes take place among the nine Supreme Court justices, and then generalize to the number of handshakes in any size group. Students explore the problem using a verbal description, a table, a graph, a picture, and an algebraic formula.

Launch/Share:

Ask students about the supreme court and what the members do each time they begin a session. Begin by asking how many handshakes does each person make. Allow students to work in groups of four to find the number of handshakes that occur with a group of nine people. Inform students that they will be presenting their solutions to the entire class, so they should keep a record of their work.

Help those who struggle by asking them to consider a simpler problem. For instance, you may ask, "In your group, how many handshakes are possible?" Help students model this by having them shake hands with one another. Acting out the problem in this way may encourage groups to combine to find the number of handshakes when there are more than four people.

The specific problem of finding the number of handshakes for the nine justices is a good way to introduce the problem, but the real value of this problem is having students generalize their results. Students who find the specific answer for nine people [36 handshakes] should be asked to keep working and find a general rule that will let them find the number of handshakes for a group of n people.

To help with this investigation, distribute the Handshake Activity Sheet.

<http://illuminations.nctm.org/uploadedFiles/Content/Lessons/Resources/6-8/Supreme-AS-Handshakes.pdf>

Ask students to discuss their findings with the class. Display lists, tables, and vertex-edge graphs for students to consider. Students may need overhead sheets or whiteboards to show their work to the class.

Summarize:

Show that there are 36 ($8 + 7 + 6 + 5 + 4 + 3 + 2 + 1$) handshakes in all.

Ask, “How many handshakes occur when there are 30 people?” Review student problem-solving methods used so far. Students will realize that list-making and vertex edge graphs are too cumbersome for the new problem and that there must be a more efficient method to get to the answer.

To help generalize this problem, work toward the equation: $n(n-1)$ as a solution to all situations.

Extension:

Have students figure out how many handshakes it would take to shake hands of all student in their class and their school.

Post Test:

- 1) If there are 9 people in a room, how many total handshakes would there be?

Lesson 7: Road Trip! (3 Days)

Adapted from class notes: Math 6200, Bemidji State University, Summer 2016

And from Illuminations website: <http://illuminations.nctm.org/Lesson.aspx?id=2721>

MN Math Standards:

8.2.2.4 Represent arithmetic sequences using equations, tables, graphs and verbal descriptions, and use them to solve problems.

8.2.2.5 Represent geometric sequences using equations, tables, graphs and verbal descriptions, and use them to solve problems.

Pre Test:

- 1) Describe how you would plan a trip that visited 5 cities that would make you drive the fewest miles?

Objective:

Students will plan a road trip, starting in Cleveland, to visit friends in Cincinnati, Pittsburgh, Baltimore, and Boston. However, with the price of gas over \$2 a gallon, they will figure out the shortest travel route to save on expenses. This lesson investigates two different methods to determine the shortest route: the Nearest Neighbor method and the Greedy Method.

Launch:

Have students share what they may have to do if they were going on a trip and wanted to save the most money on gas? Put up a picture of the map of the cities that you will be visiting.

<http://illuminations.nctm.org/uploadedFiles/Content/Lessons/Resources/9-12/RoadTrip-OH-map.pdf>

Explore/Share:

Nearest Neighbor Method:

Have students work in groups doing the Nearest Neighbor method of visiting all the cities.

<http://illuminations.nctm.org/uploadedFiles/Content/Lessons/Resources/9-12/RoadTrip-AS-NearestNeighbor.pdf>

When students are finished then have them share answers to the problems on the sheet.

Now have the students do the same problem using the Greedy method (or Cheapest Link) of visiting all the cities. Again have students work in groups doing the Greedy Method worksheet to visit all the cities.

<http://illuminations.nctm.org/uploadedFiles/Content/Lessons/Resources/9-12/RoadTrip-AS-CheapestLink.pdf>

When students are finished have them share answers with the rest of the class.

Summarize:

Review the two different methods used to solve this problem: Nearest Neighbor and Greedy . You may want to discuss with students: besides a traveling salesman career, when else is this type of problem-solving (for finding

the shortest route) useful? Which algorithm did they think was most useful and why? What factors might change their choice?

Post Test:

- 1) Describe how you would plan a trip that visits 5 cities, how would you go about planning the trip so that you would drive the least amount of miles.